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Title. The nonlocal p-Laplacian operator, the associated evolution equation

Abstract. The nonlocal p-Laplacian operator, the associated evolution equation and variational regularization, governed by a given kernel, have applications in various areas of science and engineering. In particular, they are modern tools for massive data processing (including signals, images, geometry), and machine learning tasks such as classification. In practice, however, these models are implemented in discrete form (in space and time, or in space for variational regularization) as a numerical approximation to a continuous problem, where the kernel is replaced by an adjacency matrix of a graph. Yet, few results on the consistency of these discretization are available. In particular it is largely open to determine when do the solutions of either the evolution equation or the variational problem of graph-based tasks converge (in an appropriate sense), as the number of vertices increases, to a well-defined object in the continuum setting, and if yes, at which rate. In this manuscript, we lay the foundations to address these questions. Combining tools from graph theory, convex analysis, nonlinear semigroup theory and evolution equations, we give a rigorous interpretation to the continuous limit of the discrete nonlocal p-Laplacian evolution and variational problems on graphs. More specifically, we consider a sequence of (deterministic) graphs converging to a so-called limit object known as the graphon. If the continuous p-Laplacian evolution and variational problems are properly discretized on this graph sequence, we prove that the solutions of the sequence of discrete problems converge to the solution of the continuous problem governed by the graphon, as the number of graph vertices grows to infinity. Along the way, we provide a consistency/error bounds. In turn, this allows to establish the convergence rates for different graph models. In particular, we highlight the role of the graphon geometry/regularity. For random graph sequences, using sharp deviation inequalities, we deliver nonasymptotic convergence rates in probability and exhibit the different regimes depending on p, the regularity of the graphon and the initial data.