

Chérif Amrouche (Pau)

Title. Divergence Elliptic Equations in Lipschitz and in C^1 Domains.

Abstract.

We are interested here in questions related to the study of some divergence elliptic equations in bounded Lipschitz or C^1 domains:

$$-\operatorname{div}(a\nabla u) + bu = f \quad \text{in } \Omega, \quad (1)$$

with **Dirichlet** or **Neumann** boundary condition. We will consider three different cases.

Case 1. We assume $a = 1$ and $b = 0$, corresponding to the **Laplace equation**. We will give some new results on the **traces** of non smooth functions, harmonic or non-harmonic. Using in particular the interpolation theory, we are going to study the questions of existence and **maximal regularity** of solutions in **fractional Sobolev** spaces or with **weights** associated with the **distance to the boundary**.

Case 2. We assume that $b = 0$ and a satisfies the classical condition to ensure the ellipticity of the operator $-\operatorname{div}(a \operatorname{grad})$. We will concentrate on the case of generalized solutions in $W^{1,p}(\Omega)$ with $1 < p < \infty$.

Case 3. We will finally consider the following problem:

$$-\operatorname{div}(\varrho^\alpha \nabla u) + k \frac{u}{\varrho^\beta} = f \quad \text{in } \Omega, \quad (2)$$

with or without boundary condition and where k is a non negative constant and α and β belong to the interval $[0, 1]$.

References

- [1] C. AMROUCHE AND M. MOUSSAOUI. The Dirichlet problem for the Laplacian in Lipschitz domains. Submitted. See also the abstract in <https://arxiv.org/pdf/2204.02831.pdf>
- [2] B.E.J. DAHLBERG, C.E. KENIG, J. PIPHER AND G.C. VERCHOTA. Area integral estimates for higher order elliptic equations and systems. *Ann. Inst. Fourier*, **47-5**, 1425–1461, (1997).
- [3] D. JERISON AND C.E. KENIG. The Inhomogeneous Dirichlet Problem in Lipschitz Domains, *J. Funct. Anal.* **130**, 161–219, (1995).
- [4] J.L. LIONS AND E. MAGENES. *Problèmes aux limites non-homogènes et applications*, Vol. 1, Dunod, Paris, (1969).
- [5] J. NEČAS. *Direct methods in the theory of elliptic equations*. Springer Monographs in Mathematics. Springer, Heidelberg, (2012).
- [6] G.C. VERCHOTA. The biharmonic Neumann problem in Lipschitz domains. *Acta Math.* **194-2**, 217–279, (2005).